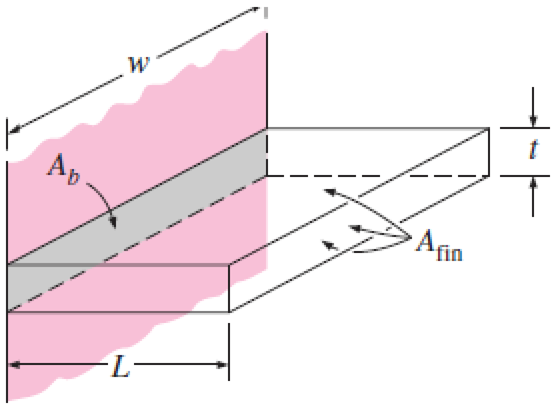
Fin Efficiency

- Finding the efficiency of the fin is important.
- Heat transfer from the fin is maximum when



(*b*) Surface with a fin

$$A_{fin} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$

$$\dot{Q}_{fin,\,max} = hA_{fin}\,(T_b - T_\infty)$$

- To account for the effect of this decrease in temperature on heat transfer, we define a **fin efficiency as**

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\,max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

or

$$\dot{Q}_{fin} = \eta_{fin}\,\dot{Q}_{fin,\,max} = \eta_{fin}\,hA_{fin}\,(T_b - T_\infty)$$

where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with insulated tips*, the fin efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL} \tag{3-70}$$

and

$$\eta_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{\tanh aL}{aL} \tag{3-71}$$

since $A_{fin} = pL$ for fins with constant cross section. Equation 3–71 can also be used for fins subjected to convection provided that the fin length L is replaced by the corrected length L_c.

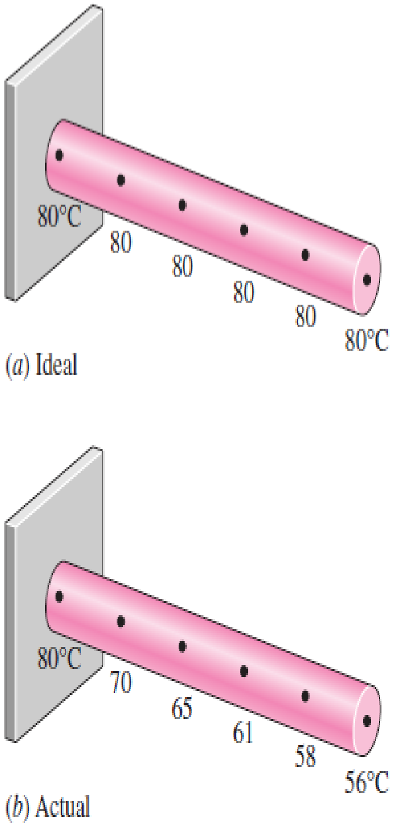


FIGURE 3–41
Ideal and actual temperature distribution in a fin.

Fin Efficiency Relationship

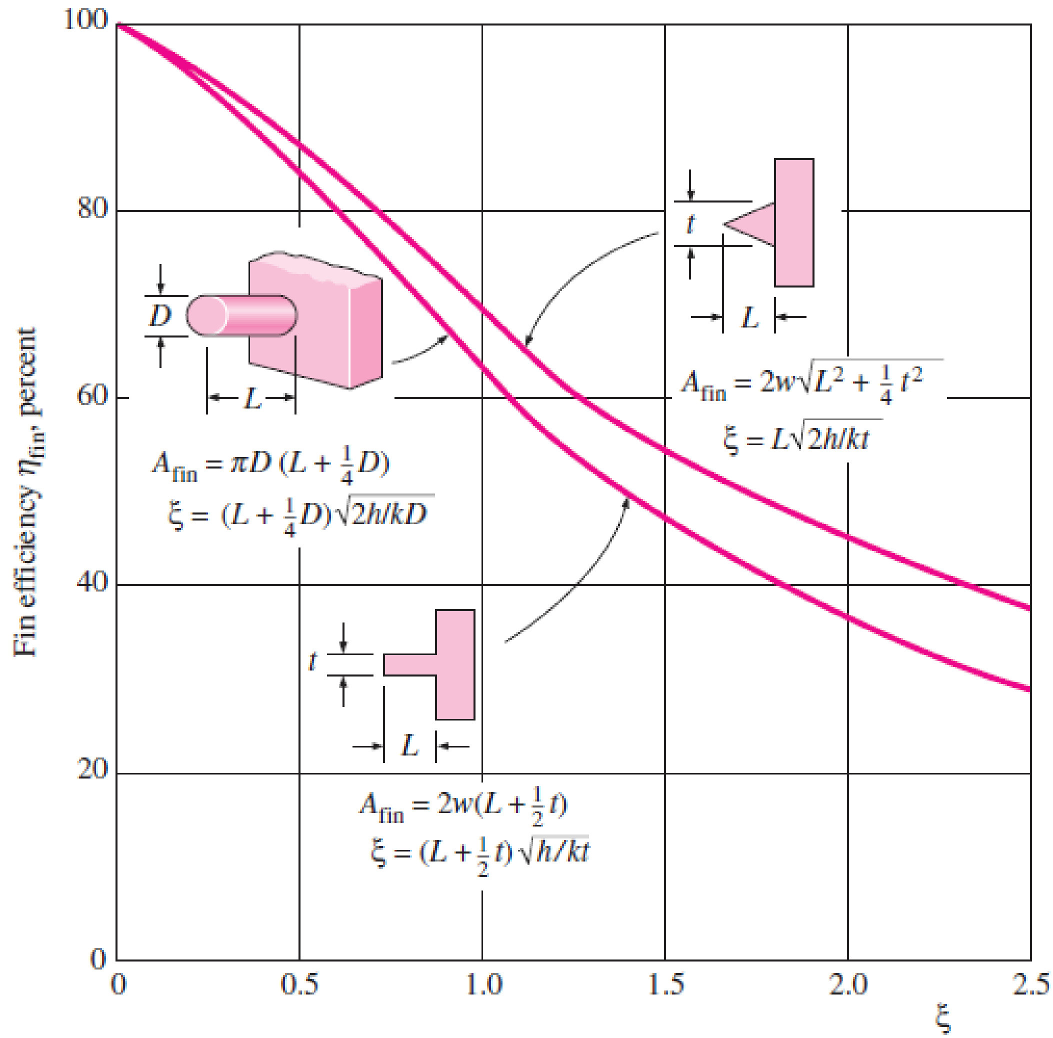


FIGURE 3–42
Efficiency of circular, rectangular, and triangular fins on a plain surface of width w (from Gardner, Ref. 6).

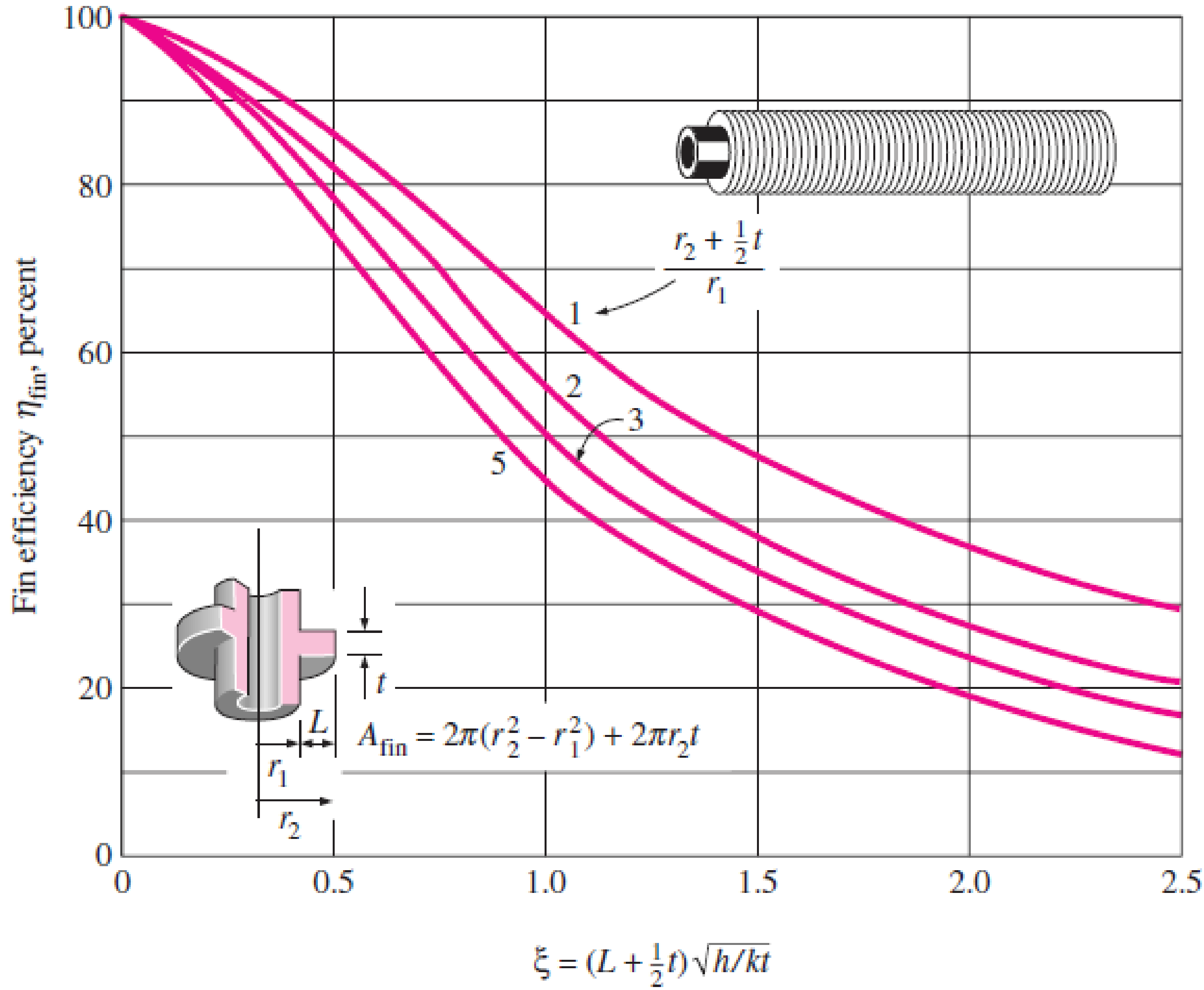


FIGURE 3–43

Efficiency of circular fins of length L and constant thickness t (from Gardner, Ref. 6).

Fin Effectiveness

Fins are used to *enhance* heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that adding fins on a surface will *enhance* heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. The performance of fins expressed in terms of the *fin effectiveness* ε_{fin} is defined as (Fig. 3–44)

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b} \quad \text{(3-72)}$$

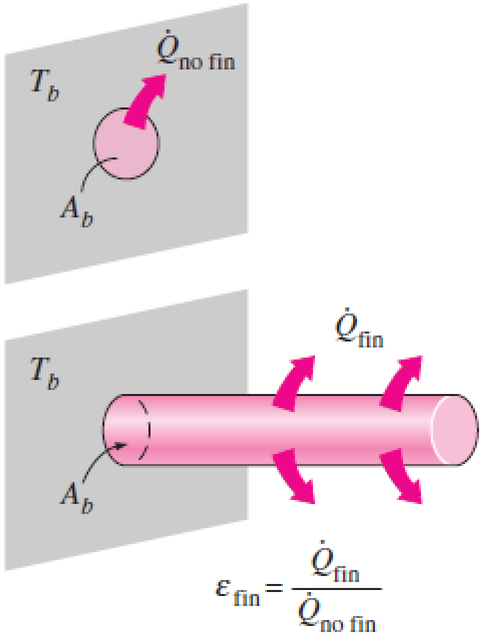


FIGURE 3–44
The effectiveness of a fin.

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. An effectiveness of $\varepsilon_{\text{fin}} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium. An effectiveness of $\varepsilon_{\text{fin}} < 1$ indicates that the fin actually acts as *insulation,* slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used. An effectiveness of $\varepsilon_{\text{fin}} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

Note that both the fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}}\, hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}} \quad \text{(3-73)}$$

The rate of heat transfer from a sufficiently *long* fin of *uniform* cross section under steady conditions is given by Eq. 3–61. Substituting this relation into Eq. 3–72, the effectiveness of such a long fin is determined to be

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \tag{3-74}$$

since $A_c = A_b$ in this case. We can draw several important conclusions from the fin effectiveness relation above for consideration in the design and selection of the fins:

- The *thermal conductivity* k of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.
- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. This criterion is satisfied by *thin* plate fins and *slender* pin fins.
- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient*. Thus, the use of fins is more easily justified when the medium is a *gas* instead of a liquid and the heat transfer is by *natural convection* instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the *gas* side.

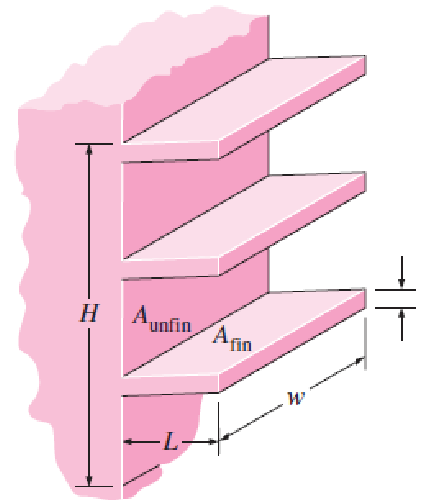
When determining the rate of heat transfer from a finned surface, we must consider the *unfinned portion* of the surface as well as the *fins*. Therefore, the rate of heat transfer for a surface containing n fins can be expressed as

$$\begin{aligned}\dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\ &= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty) \qquad \textbf{(3-75)}\end{aligned}$$

We can also define an **overall effectiveness** for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)} \qquad \textbf{(3-76)}$$

where $A_{\text{no fin}}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface (Fig. 3–45). Note that the overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.



$A_{\text{no fin}} = w \times H$
$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$
$A_{\text{fin}} = 2 \times L \times w + t \times w$ (one fin)
$\approx 2 \times L \times w$

FIGURE 3–45
Various surface areas associated with a rectangular surface with three fins.

Proper Length of a Fin

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is

Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL \qquad \textbf{(3-77)}$$

Specially designed finned surfaces called *heat sinks,* which are commonly used in the cooling of electronic equipment, involve one-of-a-kind complex geometries, as shown in Table 3–4. The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances* R in °C/W, which is defined as

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_\infty}{R} = hA_{\text{fin}}\,\eta_{\text{fin}}\,(T_b - T_\infty) \qquad \textbf{(3-78)}$$

A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

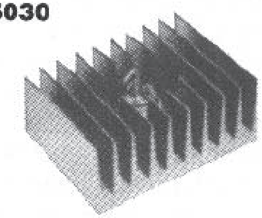
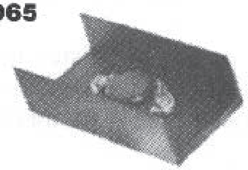
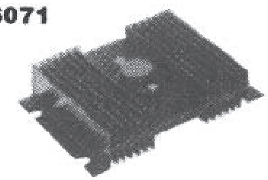
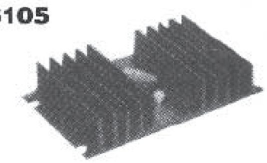
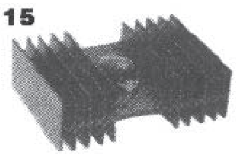
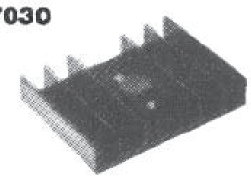
TABLE 3–3

The variation of heat transfer from a fin relative to that from an infinitely long fin

aL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

TABLE 3–4

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in.) long (courtesy of Vemaline Products, Inc.).

HS 5030		R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm^2
HS 6065		R = 5°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm^2
HS 6071		R = 1.4°C/W (vertical) R = 1.8°C/W (horizontal) Dimensions: 76 mm × 92 mm × 26 mm Surface area: 968 cm^2
HS 6105		R = 1.8°C/W (vertical) R = 2.1°C/W (horizontal) Dimensions: 76 mm × 127 mm × 91 mm Surface area: 677 cm^2
HS 6115		R = 1.1°C/W (vertical) R = 1.3°C/W (horizontal) Dimensions: 76 mm × 102 mm × 25 mm Surface area: 929 cm^2
HS 7030		R = 2.9°C/W (vertical) R = 3.1°C/W (horizontal) Dimensions: 76 mm × 97 mm × 19 mm Surface area: 290 cm^2

EXAMPLE 3–10 Maximum Power Dissipation of a Transistor

Power transistors that are commonly used in electronic devices consume large amounts of electric power. The failure rate of electronic components increases almost exponentially with operating temperature. As a rule of thumb, the failure rate of electronic components is halved for each 10°C reduction in the junction operating temperature. Therefore, the operating temperature of electronic components is kept below a safe level to minimize the risk of failure.

The sensitive electronic circuitry of a power transistor at the junction is protected by its case, which is a rigid metal enclosure. Heat transfer characteristics of a power transistor are usually specified by the manufacturer in terms of the case-to-ambient thermal resistance, which accounts for both the natural convection and radiation heat transfers.

The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 10 W is given to be 20°C/W. If the case temperature of the transistor is not to exceed 85°C, determine the power at which this transistor can be operated safely in an environment at 25°C.

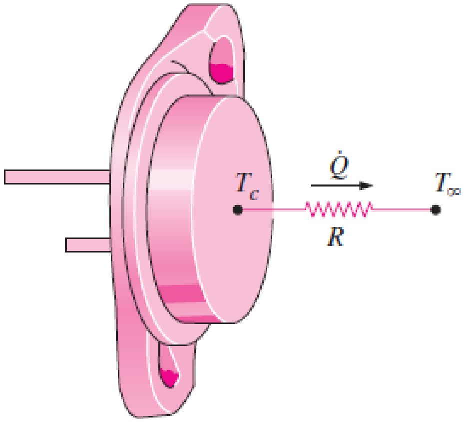


FIGURE 3–47
Schematic for Example 3–10.

SOLUTION The maximum power rating of a transistor whose case temperature is not to exceed 85°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 85°C.

Properties The case-to-ambient thermal resistance is given to be 20°C/W.

Analysis The power transistor and the thermal resistance network associated with it are shown in Fig. 3–47. We notice from the thermal resistance network that there is a single resistance of 20°C/W between the case at T_c = 85°C and the ambient at T_∞ = 25°C, and thus the rate of heat transfer is

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{\text{case-ambient}} = \frac{T_c - T_\infty}{R_{\text{case-ambient}}} = \frac{(85 - 25)°\text{C}}{20°\text{C/W}} = 3\ \text{W}$$

Therefore, this power transistor should not be operated at power levels above 3 W if its case temperature is not to exceed 85°C.

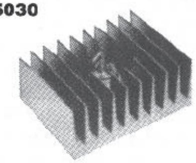
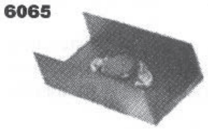
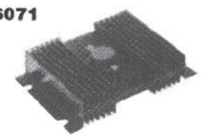
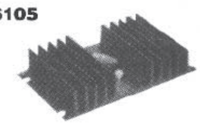
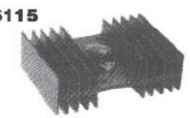
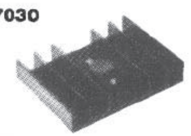
Discussion This transistor can be used at higher power levels by attaching it to a heat sink (which lowers the thermal resistance by increasing the heat transfer surface area, as discussed in the next example) or by using a fan (which lowers the thermal resistance by increasing the convection heat transfer coefficient).

EXAMPLE 3–11 Selecting a Heat Sink for a Transistor

A 60-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 30°C.

TABLE 3–4

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in.) long (courtesy of Vemaline Products, Inc.).

HS 5030 	R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm^2
HS 6065 	R = 5°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm^2
HS 6071 	R = 1.4°C/W (vertical) R = 1.8°C/W (horizontal) Dimensions: 76 mm × 92 mm × 26 mm Surface area: 968 cm^2
HS 6105 	R = 1.8°C/W (vertical) R = 2.1°C/W (horizontal) Dimensions: 76 mm × 127 mm × 91 mm Surface area: 677 cm^2
HS 6115 	R = 1.1°C/W (vertical) R = 1.3°C/W (horizontal) Dimensions: 76 mm × 102 mm × 25 mm Surface area: 929 cm^2
HS 7030 	R = 2.9°C/W (vertical) R = 3.1°C/W (horizontal) Dimensions: 76 mm × 97 mm × 19 mm Surface area: 290 cm^2

SOLUTION A commercially available heat sink from Table 3–4 is to be selected to keep the case temperature of a transistor below 90°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The rate of heat transfer from a 60-W transistor at full power is $\dot{Q} = 60$ W. The thermal resistance between the transistor attached to the heat sink and the ambient air for the specified temperature difference is determined to be

$$\dot{Q} = \frac{\Delta T}{R} \longrightarrow R = \frac{\Delta T}{\dot{Q}} = \frac{(90 - 30)°\text{C}}{60 \text{ W}} = 1.0°\text{C/W}$$

Therefore, the thermal resistance of the heat sink should be below 1.0°C/W. An examination of Table 3–4 reveals that the HS 5030, whose thermal resistance is 0.9°C/W in the vertical position, is the only heat sink that will meet this requirement.

EXAMPLE 3–12 Effect of Fins on Heat Transfer from Steam Pipes

Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3$ cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins ($k = 180$ W/m · °C) of outer diameter $D_2 = 6$ cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. 3–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25$°C, with a combined heat transfer coefficient of $h = 60$ W/m² · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

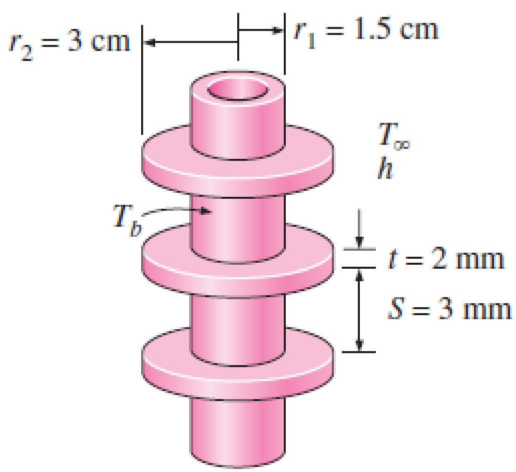


FIGURE 3–48
Schematic for Example 3–12.

SOLUTION Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 180$ W/m · °C.

Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\text{ m})(1\text{ m}) = 0.0942\text{ m}^2$$

$$\begin{aligned}\dot{Q}_{\text{no fin}} &= hA_{\text{no fin}}(T_b - T_\infty)\\ &= (60\text{ W/m}^2\cdot{}^\circ\text{C})(0.0942\text{ m}^2)(120 - 25)^\circ\text{C}\\ &= 537\text{ W}\end{aligned}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\left.\begin{aligned} \frac{r_2 + \frac{1}{2}t}{r_1} &= \frac{(0.03 + \frac{1}{2} \times 0.002)\text{ m}}{0.015\text{ m}} = 2.07 \\ (L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} &= (0.015 + \tfrac{1}{2} \times 0.002)\text{ m} \times \sqrt{\frac{60\text{ W/m}^2 \cdot {}^\circ\text{C}}{(180\text{ W/m} \cdot {}^\circ\text{C})(0.002\text{ m})}} = 0.207 \end{aligned}\right\} \eta_{\text{fin}} = 0.95$$

$$\begin{aligned} A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\ &= 2\pi[(0.03\text{ m})^2 - (0.015\text{ m})^2] + 2\pi(0.03\text{ m})(0.002\text{ m}) \\ &= 0.00462\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_\infty) \\ &= 0.95(60\text{ W/m}^2 \cdot {}^\circ\text{C})(0.00462\text{ m}^2)(120 - 25)^\circ\text{C} \\ &= 25.0\text{ W} \end{aligned}$$

Heat transfer from the unfinned portion of the tube is

$$\begin{aligned} A_{\text{unfin}} &= \pi D_1 S = \pi(0.03\text{ m})(0.003\text{ m}) = 0.000283\text{ m}^2 \\ \dot{Q}_{\text{unfin}} &= h A_{\text{unfin}}(T_b - T_\infty) \\ &= (60\text{ W/m}^2 \cdot {}^\circ\text{C})(0.000283\text{ m}^2)(120 - 25)^\circ\text{C} \\ &= 1.60\text{ W} \end{aligned}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6) \text{ W} = 5320 \text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783 \text{ W}} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320 \text{ W}}{537 \text{ W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

Heat Transfer in Common Configurations

An important class of heat transfer problems for which simple solutions are obtained encompasses those involving two surfaces maintained at *constant* temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$Q = Sk(T_1 - T_2) \tag{3-79}$$

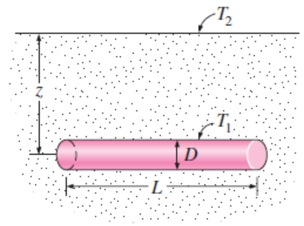
where S is the **conduction shape factor,** which has the dimension of *length*, and k is the thermal conductivity of the medium between the surfaces. The conduction shape factor depends on the *geometry* of the system only.

TABLE 3-5

Conduction shape factors S for several configurations for use in $\dot{Q} = kS(T_1 - T_2)$ to determine the steady rate of heat transfer through a medium of thermal conductivity k between the surfaces at temperatures T_1 and T_2

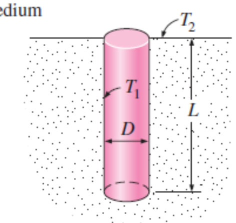
(1) Isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$ and $z > 1.5D$)

$$S = \frac{2\pi L}{\ln(4z/D)}$$



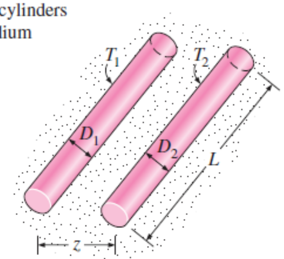
(2) Vertical isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$)

$$S = \frac{2\pi L}{\ln(4L/D)}$$



(3) Two parallel isothermal cylinders placed in an infinite medium ($L >> D_1, D_2, z$)

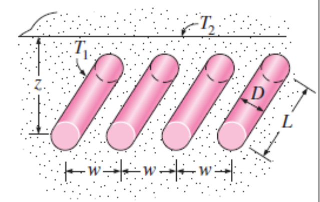
$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$



(4) A row of equally spaced parallel isothermal cylinders buried in a semi-infinite medium ($L >> D, z$ and $w > 1.5D$)

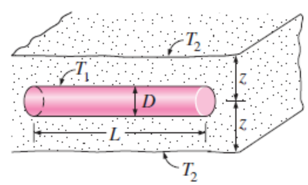
$$S = \frac{2\pi L}{\ln\left(\frac{2w}{\pi D}\sinh\frac{2\pi z}{w}\right)}$$

(per cylinder)



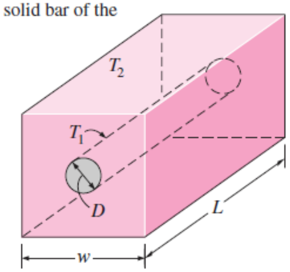
(5) Circular isothermal cylinder of length L in the midplane of an infinite wall ($z > 0.5D$)

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$



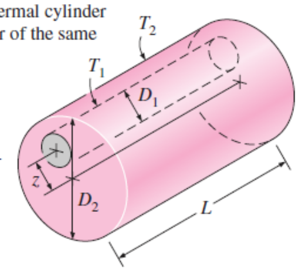
(6) Circular isothermal cylinder of length L at the center of a square solid bar of the same length

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$



(7) Eccentric circular isothermal cylinder of length L in a cylinder of the same length ($L > D_2$)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$$



(8) Large plane wall

$$S = \frac{A}{L}$$

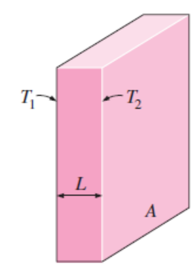
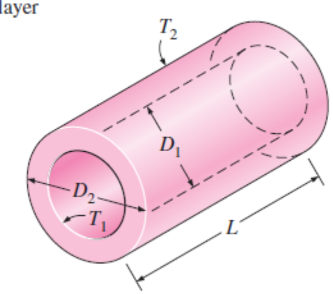


TABLE 3-5 (CONCLUDED)

(9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln (D_2/D_1)}$$



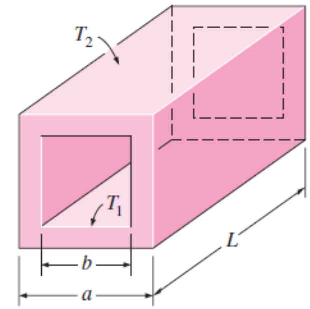
(10) A square flow passage

(*a*) For $a/b > 1.4$,

$$S = \frac{2\pi L}{0.93 \ln (0.948\, a/b)}$$

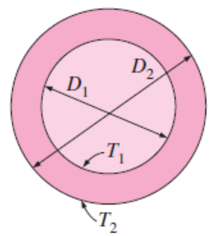
(*b*) For $a/b < 1.41$,

$$S = \frac{2\pi L}{0.785 \ln (a/b)}$$



(11) A spherical layer

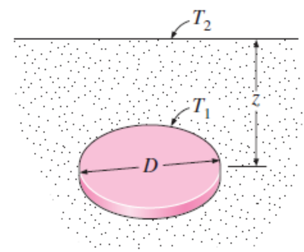
$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$



(12) Disk buried parallel to the surface in a semi-infinite medium ($z >> D$)

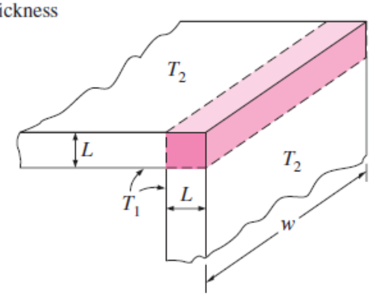
$$S = 4D$$

($S = 2D$ when $z = 0$)



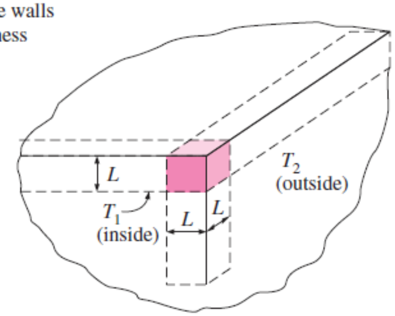
(13) The edge of two adjoining walls of equal thickness

$$S = 0.54w$$



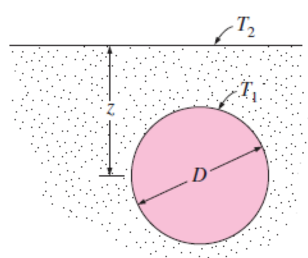
(14) Corner of three walls of equal thickness

$$S = 0.15L$$



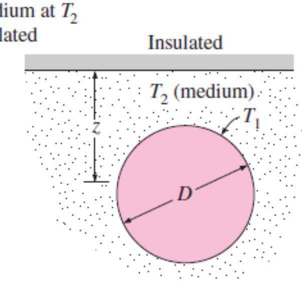
(15) Isothermal sphere buried in a semi-infinite medium

$$S = \frac{2\pi D}{1 - 0.25D/z}$$



(16) Isothermal sphere buried in a semi-infinite medium at T_2 whose surface is insulated

$$S = \frac{2\pi D}{1 + 0.25D/z}$$



EXAMPLE 3–13 Heat Loss from Buried Steam Pipes

A 30-m-long, 10-cm-diameter hot water pipe of a district heating system is buried in the soil 50 cm below the ground surface, as shown in Figure 3–49. The outer surface temperature of the pipe is 80°C. Taking the surface temperature of the earth to be 10°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.

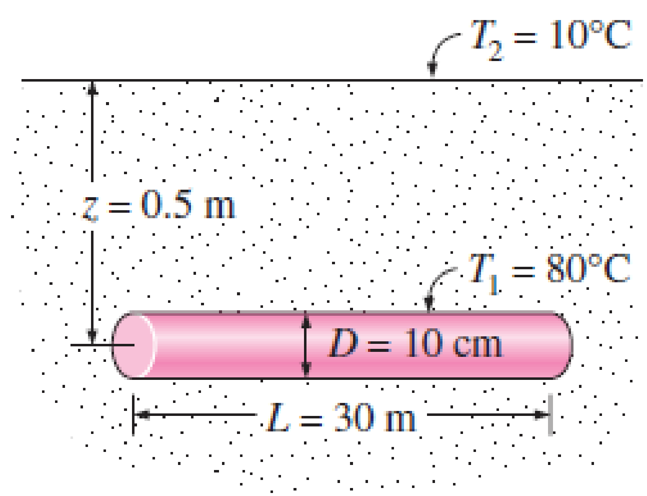


FIGURE 3–49
Schematic for Example 3–13.

SOLUTION The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be $k = 0.9$ W/m · °C.

Analysis The shape factor for this configuration is given in Table 3–5 to be

$$S = \frac{2\pi L}{\ln(4z/D)}$$

since $z > 1.5D$, where z is the distance of the pipe from the ground surface, and D is the diameter of the pipe. Substituting,

$$S = \frac{2\pi \times (30 \text{ m})}{\ln(4 \times 0.5/0.1)} = 62.9 \text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (62.9 \text{ m})(0.9 \text{ W/m} \cdot \text{°C})(80 - 10)\text{°C} = \mathbf{3963 \text{ W}}$$

Discussion Note that this heat is conducted from the pipe surface to the surface of the earth through the soil and then transferred to the atmosphere by convection and radiation.

EXAMPLE 3–14 Heat Transfer between Hot and Cold Water Pipes

A 5-m-long section of hot and cold water pipes run parallel to each other in a thick concrete layer, as shown in Figure 3–50. The diameters of both pipes are 5 cm, and the distance between the centerline of the pipes is 30 cm. The surface temperatures of the hot and cold pipes are 70°C and 15°C, respectively. Taking the thermal conductivity of the concrete to be $k = 0.75$ W/m · °C, determine the rate of heat transfer between the pipes.

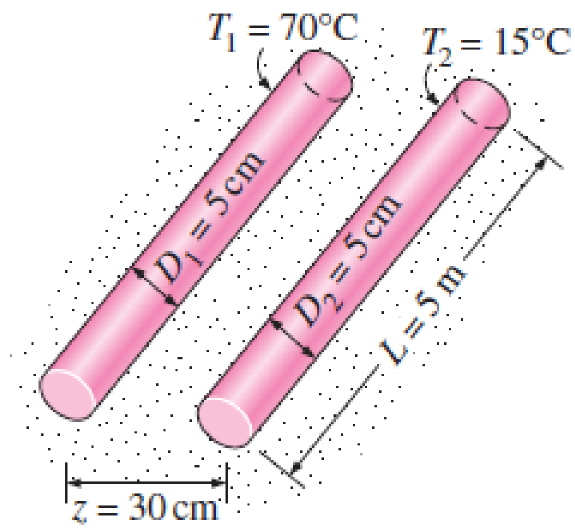


FIGURE 3–50
Schematic for Example 3-14.

SOLUTION Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be $k = 0.75$ W/m · °C.

Analysis The shape factor for this configuration is given in Table 3–5 to be

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$

where z is the distance between the centerlines of the pipes and L is their length. Substituting,

$$S = \frac{2\pi \times (5 \text{ m})}{\cosh^{-1}\left(\dfrac{4 \times 0.3^2 - 0.05^2 - 0.05^2}{2 \times 0.05 \times 0.05}\right)} = 6.34 \text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

$$\dot{Q} = Sk(T_1 - T_2) = (6.34 \text{ m})(0.75 \text{ W/m} \cdot \text{°C})(70 - 15\text{°})\text{C} = 262 \text{ W}$$

Discussion We can reduce this heat loss by placing the hot and cold water pipes further away from each other.